volve an infinite acquired velocity, which is absurd." In this way Peirce justifies Galileo's conclusion.

The error in Peirce's reasoning seems to me perfectly apparent. When $s=0$ he takes C to be an infinitesimal, while really C is an absolute zero. When s and t are both zero, e^{at} is not zero, hence e^{at}, multiplied by an infinitesimal, cannot be equal to absolute zero. An infinitesimal is a variable whose limit is zero, but the variable never reaches its limit. If we consider an infinitesimal an extremely small quantity, we must still remember that it is a quantity. Now, if C is absolute zero, then s can never be different from zero, no matter how large e^{at} may be.

It is easy to illustrate my conclusion by physical examples. A particle is placed in a smooth tube which revolves horizontally about an axis through its center. With what velocity will the particle move? The only force impelling the particle along the tube is the centrifugal force due to rotation. Hence we have $\dfrac{d^2s}{dt^2}=w^2s$ and $\dfrac{ds}{dt}=ws$, where w is the uniform angular velocity. Here the velocity is proportional to the distance from the axis. Suppose now that the particle lies initially at rest *in the axis*. Will it begin to move? There is no reason why it should move one way any more than the other.

The two assumptions in our problem are contraries. The first excludes the possibility of motion; the second declares that motion exists. From assumptions that are contraries no conclusion can be drawn.

DIOPHANTINE ANALYSIS.

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80. **Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.**

Find three square numbers whose reciprocals form an arithmetical progression.

I. **Solution by the PROPOSER.**

When three numbers are in arithmetical progression, the products of these numbers, taken two at a time, will give three numbers whose *reciprocals* are in arithmetical progression.

Let $x-a$, x, and $x+a$=three numbers in arithmetical progression. Then $x(x-a)$, $(x-a)(x+a)$, and $x(x+a)$=the three numbers whose reciprocals are in arithmetical progression.

$$\text{For, } \frac{1}{x(x-a)}-\frac{1}{(x-a)(x+a)}=\frac{1}{(x-a)(x+a)}-\frac{1}{x(x+a)}=\frac{a}{x(x-a)(x+a)}.$$

The general values for three *squares* in arithmetical progression are $(p^2-q^2-2pq)^2$, $(p^2+q^2)^2$, and $(p^2-q^2+2pq)^2$, where $x=(p^2+q^2)^2$, and a the common difference$=4pq(p^2-q^2)$.

Put $p=2$ and $q=1$; then 1^2, 5^2, and 7^2 are three squares in arithmetical progression. Whence the three squares whose reciprocals form an arithmetical

progression are $5^2=1^2\times5^2$, $7^2=1^2\times7^2$, and $35^2=5^2\times7^2$; and the progression is $\frac{1}{25}$, $\frac{1}{49}$, $\frac{1}{1225}$.

Put $p=3$ and $q=2$; then the required progression is $(\frac{1}{91})^2$, $(\frac{1}{119})^2$, $(\frac{1}{221})^2$.

II. **Solution by O. S. WESTCOTT, A. M., Sc. D., Maywood, Ill.**

Let x^2, y^2, z^2 represent the numbers. Then $1/z^2-1/y^2=1/y^2-1/x^2$. And $1/z^2+1/x^2=2/y^2$, or $y^2(x^2+z^2)=2x^2z^2$, or $y^2/x^2z^2=2/(x^2+z^2)$.

Since the first member of this equation is a square, the second must be.

$2/(x^2+z^2)=4[2(x^2+z^2)]$, and we have to make $2(x^2+z^2)$ a square.

Put $x=7$ and $z=1$; then $2(x^2+z^2)=2(49+1)=10^2$. Hence the numbers are 49, $\frac{49}{25}$, 1; the progression being $\frac{1}{49}$, $\frac{25}{49}$, 1.

Or put $x=\frac{1}{7}$ and $z=1$; then $2(x^2+z^2)=2[(\frac{1}{49})+1]=(\frac{10}{7})^2$, and the numbers are $\frac{1}{49}$, $\frac{1}{25}$, 1; the progression being 49, 25, 1.

III. **Solution by SYLVESTER ROBINS, North Branch, N. J.**

Let a^2, x^2 and b^2 represent three squares whose reciprocals $1/a^2$, $1/x^2$, and $1/b^2$ are in arithmetical progression.

Then $1/a^2+1/b^2=2/x^2$, and $x^2=2a^2b^2/(a^2+b^2)$, a square.

Expand $\sqrt{2}=1$, $\frac{7}{5}$, $\frac{41}{29}$, $\frac{239}{169}$, $\frac{1393}{985}$, $\frac{8119}{5741}$, etc.

Say $a=1$; $b=7$, 41, 239, 1393, 8119, etc.

Then 1^2, $(2\times1^2\times7^2)/(1^2+7^2)$, 7^2 .....1, $\frac{49}{25}$, 49.

1^2, $(2\times1^2\times41^2)/(1^2+41^2)$, 41^2......1, $\frac{1681}{841}$, 1681.

1^2, $(2\times1^2\times239^2)/(1^2+239^2)$, 239^2......1, $\frac{57121}{28561}$, 57121.

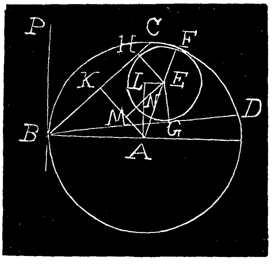
COOPER D. SCHMITT and *G. B. M. ZERR* refer to Problem 78. See solutions of that problem in MONTHLY for March, pages 82—83, by *CHARLES C. CROSS*, *JOSIAH H. DRUMMOND*, *M. A. GRUBER*, and *G. B. M. ZERR*, who also solved the above problem.

AVERAGE AND PROBABILITY.

84. **Proposed by L. C. WALKER, Associate Professor of Mathematics, Leland Stanford, Jr., University, Palo Alto, Cal.**

From a point in the circumference of a circle two chords are drawn; find (1) the average radius, and (2) the average area of the circle which touches the two chords and the given circle.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.



Let $AB=AF=r$, $HE=GE=FE=x$, $\angle PBD=\theta$, $\angle PBC=\varphi$.

Then $BD=2r\sin\theta$, $BC=2r\sin\varphi$, $AK=r\cos\varphi$, $AN=r\cos\theta$, $BH=BG=BK+KH=BN+NG$.

$KH=ME=\sqrt{[(r-x)^2-(r\cos\varphi-x)^2]}$
$\quad=\sqrt{[r^2\sin^2\varphi-2rx(1-\cos\varphi)]}$.

$NG=LE=\sqrt{[(r-x)^2-(r\cos\theta+x)^2]}$
$\quad=\sqrt{[r^2\sin^2\theta-2rx(1+\cos\theta)]}$.

$\therefore\ r\sin\varphi+\sqrt{[r^2\sin^2\varphi-2rx(1-\cos\varphi)]}$